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Non-commutative geometry from string theory: annulus corrections

Mark Laidlaw

University of British Columbia
6224 Agricultural Road
Vancouver BC, Canada
E-mail: laidlaw@physics.ubc.ca

Abstract: We develop a method in which it is possible to calculate one loop corrections to the noncommutativity parameter found for open strings in a background $F_{\mu\nu}$ field. We first reproduce the well-known disk results for $\theta^{\mu\nu}$. We then consider the case of charged and neutral open strings on the brane, and show in both cases that the result is the same as in the disk case, apart from a multiplicative factor due to the open string tachyon. We also consider the case of open strings stretched between two-branes and show that our method reproduces known disk results.

Keywords: Bosonic Strings, D-branes.

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Contents

1. Introduction

In the past few years there has been much activity exploring the stringy origins of noncommutative geometry. [1]–[5] Most of this activity has been done by considering a bosonic string in an antisymmetric background with the disk topology and examining the corresponding commutators of the X^μ fields. Broadly speaking there are two approaches in the literature; one is to find a mode expansion for the X fields which is orthogonal and consistent with the boundary conditions induced by the background field, and after calculating the commutator of the mode coefficients using this to work out the full commutator between the Xs [4]–[7], while the other is to calculate an exact propagator for the bosonic excitations in the background and directly from that read off the commutator [1]. Both of these methods are effective, and give the same answer for the noncommutativity induced by the background field. However, neither generalizes well to higher topologies, such as the annulus. In the case of the operator approach, it relies on particular assumptions of similar block diagonal structures for the gauge fields on different branes, while exact propagators in the presence of an $F_{\mu\nu}$ background are complicated for higher genus topology. [8, 9].

With this in mind, we offer a slightly different way of deriving the noncommutativity parameter for a bosonic string in an antisymmetric background. We show that this method of calculation reproduces the known results for the disk, and proceed to calculate annulus level corrections to non-commutative geometry via an obvious generalization. While many aspects of the annulus geometry are discussed in the literature, for example [10, 11, 12], this method produces new higher order corrections. The procedure is as follows: since the lagrangian for a string in a background field is

$$S = \frac{-1}{4\pi\alpha'}\int d\sigma d\tau \sqrt{-g} g_{ab}\partial^a X^\mu \partial^b X_\mu + \frac{1}{2}\int_{\partial\Sigma} d\tau F_{\mu\nu} X^\nu \partial_\tau X^\mu|_{\sigma=a} \tag{1.1}$$

we treat the interaction with $F_{\mu\nu}$ as a perturbation, and find the modification to the propagator due to N interactions with this background. We then perform the sum over N to obtain an exact propagator and then using the prescription, due to Seiberg and Witten, [1], that

$$[X^\mu(\tau), X^\nu(\tau)] = \lim_{\epsilon\to 0}\mathcal{T}\Big(X^\mu(\tau)X^\nu(\tau-\epsilon) - X^\mu(\tau)X^\nu(\tau+\epsilon)\Big), \tag{1.2}$$

where $\mathcal{T}$ represents time ordering, we calculate the commutator, and find a non-trivial relation.

2. The disk

In the case of the disk it is well known that the free Green's function is [13]

$$G^{\mu\nu}(z,z') = -\alpha' \ln|z-z'|\left|z-\bar{z}'^{-1}\right|\delta^{\mu\nu}. \tag{2.1}$$

Using the parameterization $z=\rho e^{i\phi}$, with $\rho \in (0,1)$ and $\phi \in (0,2\pi)$ the propagator can be re-written on the boundary as

$$G^{\mu\nu}(e^{i\phi}, e^{i\phi'}) = 2\alpha' \sum_{m=1}^{\infty} \frac{\cos m(\phi-\phi')}{m}\delta^{\mu\nu}. \tag{2.2}$$

Keeping in mind that the interaction term is

$$L_{\text{int}} = \frac{1}{2}\int d\phi F_{\mu\nu} X^\nu \partial_\phi X^\mu, \tag{2.3}$$

it is clear that the contribution to the propagator due to N interactions with the background field is

$$\begin{aligned} G_N^{\mu\nu}(z,z') = \left(F^N\right)^{\mu\nu} \int d\theta_1 \cdots d\theta_N G(\rho e^{i\phi}, e^{i\theta_1}) \times \\ \times\, \partial_{\theta_1} G(e^{i\theta_1}, e^{i\theta_2}) \cdots \partial_{\theta_N} G(e^{i\theta_N}, \rho' e^{i\phi'}). \end{aligned} \tag{2.4}$$

Using (2.1) and (2.2) in the above, and noting that the integrals become trivial, we find that

$$G_N^{\mu\nu}(z,z') = \alpha' \left((-2\pi\alpha' F)^N\right)^{\mu\nu} \sum_{m=1}^{\infty} \frac{\rho^m}{m}(\rho'^m + \rho'^{-m}) \begin{Bmatrix} \cos m(\phi-\phi') \ N \text{ even} \\ \sin m(\phi-\phi') \ N \text{ odd} \end{Bmatrix}. \quad (2.5)$$

So, summing over N, the full propagator taking into account the background field is

$$\begin{aligned} G^{\mu\nu}(z,z') &= \alpha' \left(\frac{1}{1-(2\pi\alpha' F)^2}\right)^{\mu\nu} \sum_{m=1}^{\infty} \frac{(\rho\rho')^m + (\rho/\rho')^m}{m} \cos m(\phi-\phi') + \\ &+ \alpha' \left(\frac{-2\pi\alpha' F}{1-(2\pi\alpha' F)^2}\right)^{\mu\nu} \sum_{m=1}^{\infty} \frac{(\rho\rho')^m + (\rho/\rho')^m}{m} \sin m(\phi-\phi'). \quad (2.6) \end{aligned}$$

Now, applying this to (1.2) we see that

$$[X^\mu, X^\nu] = \lim_{\epsilon\to 0} 2\alpha' \left(\frac{-2\pi\alpha' F}{1-(2\pi\alpha' F)^2}\right)^{\mu\nu} \sum_{m=1}^{\infty} \frac{(\rho\rho')^m + (\rho/\rho')^m}{m} \sin m\epsilon. \quad (2.7)$$

In the case that ρ and ρ' are not both on the boundary (where they would be equal to 1) the $\rho\rho'$ term converges to something proportional to $\tan^{-1}(\frac{2\rho\rho' \sin\epsilon}{1-(\rho\rho')^2})$, which vanishes in the limit $\epsilon \to 0$, with something analogous happening to the other term. Thus, we see that the only non-vanishing contribution comes from $\rho = \rho' = 1$, and so in that case

$$[X^\mu, X^\nu] = \lim_{\epsilon\to 0} 2\alpha' \left(\frac{-2\pi\alpha' F}{1-(2\pi\alpha' F)^2}\right)^{\mu\nu} \sum_{m=1}^{\infty} \frac{2\sin m\epsilon}{m}. \quad (2.8)$$

Now, using $2\sum_m \frac{\sin mx}{m} = \pi - x$, this gives

$$[X^\mu, X^\nu] = 2\pi\alpha' \left(\frac{-2\pi\alpha' F}{1-(2\pi\alpha' F)^2}\right)^{\mu\nu} \quad (2.9)$$

as the commutator for the coordinates on the D-brane with a constant $F_{\mu\nu}$ field on it, in agreement with previous results.

3. The annulus

For the annulus, more care is needed, but we proceed in analogy with the above development. There are, in general, two situations that must be considered, the first is the case where both ends of the open string end on the same D-brane, in which case it provides a one loop correction to the noncommutativity parameter calculated previously, and the second is where each end is on a distinct D-brane.

The free Greens function for a boson on an annulus of inner radius a, and circumference 2π can be shown to be

$$G^{\mu\nu}(\rho e^{i\phi},\rho' e^{i\phi'}) = \frac{2\alpha'}{\ln a}\left(\sum_{m>0,n}\cos\frac{n\pi\ln\rho}{\ln a}\cos\frac{n\pi\ln\rho'}{\ln a}\frac{\cos m(\phi-\phi')}{m^2+(n\pi/\ln a)^2}+\right.$$
$$\left.+\sum_{n>0}\cos\frac{n\pi\ln\rho}{\ln a}\cos\frac{n\pi\ln\rho'}{\ln a}\left(\frac{\ln a}{n\pi}\right)^2\right)\delta^{\mu\nu}\,, \tag{3.1}$$

which is equivalent under the identification $z=\rho e^{i\phi}$ to the Greens function for the annulus given in [8, 14].

Now, using a technique that follows [13] we note that for z and z' both on the boundary of the annulus, the Greens function can be compactly expressed as

$$G^{\mu\nu}(\phi,\phi') = 2\alpha'\sum_{m=1}^{\infty}\frac{1}{m}G_m\cos m(\phi-\phi')\delta^{\mu\nu}\,, \tag{3.2}$$

where

$$G_m = \begin{pmatrix} A_m & B_m \\ B_m & A_m\end{pmatrix}, \qquad A_m = \frac{1+a^{2m}}{1-a^{2m}}\,, \qquad B_m = \frac{2a^m}{1-a^{2m}}\,. \tag{3.3}$$

Similarly, the propagator between an arbitrary point on the annulus, z, and a point on either edge, parameterized by ϕ, can be given by the row vector

$$G^{\mu\nu}(\rho e^{i\phi},\phi') = \frac{2\alpha'}{\ln a}\sum_{m=1}^{\infty}\sum_{n}\frac{1}{m^2+(n\pi/\ln a)^2}\times$$
$$\times\left(\cos\frac{n\pi\ln\rho}{\ln a},\cos\frac{n\pi\ln\rho}{\ln a}(-1)^n\right)\delta^{\mu\nu}\,, \tag{3.4}$$

and interchanging the arguments gives the transpose of this. Note that in this language, the interaction is given by

$$L_{\rm int} = \frac{1}{2}\int d\phi\Omega_{\mu\nu}X^{\nu}\partial_{\phi}X^{\mu}\,, \qquad \Omega_{\mu\nu} = \begin{pmatrix} F_{1\mu\nu} & 0 \\ 0 & F_{2\mu\nu}\end{pmatrix}, \tag{3.5}$$

where F_1 and F_2 are the field strengths at the distinct ends of the brane.

We first consider the case of an annulus diagram for a string attached at both ends to the same D-brane. The interaction of a bosonic string with a background field is equivalent to a Wilson loop inserted at its boundaries [15] and since the background is a constant field the orientation of the Wilson loop determines the sign of the charge at the endpoints of the string. There are apparently two inequivalent scenarios for this topology, the loops could be identically or oppositely oriented, corresponding to the cases of charged and neutral strings.

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3.1 Annulus corrections: charged string

The charged string corresponds to the case of identically oriented Wilson loops, and therefore, we have $F_1 = F_2 = F$. It is also important to note that there are contributions to the commutator from both ends of the string, and between opposite ends, since they all end on the same brane.

As in the case of the disk, we calculate the correction to the propagator due to N interactions with the background, and find it to be

$$\begin{aligned} G_N^{\mu\nu}(\rho e^{i\phi}, \rho' e^{i\phi'}) &= (-2\alpha'\pi)^N \frac{2\alpha'}{\ln^2 a} \left(F^N\right)^{\mu\nu} \sum_{m=1}^{\infty} \frac{1}{m} \left\{ \begin{array}{c} \sin m(\phi-\phi') \ N \text{ odd} \\ \cos m(\phi-\phi') \ N \text{ even} \end{array} \right\} \times \\ &\times \sum_n \frac{1}{m^2 + (n\pi/\ln a)^2} \left(\cos\frac{n\pi \ln\rho}{\ln a}, \cos\frac{n\pi\ln\rho}{\ln a}(-1)^n \right) G_m^{N-1} \times \\ &\times \sum_{n'} \frac{1}{m^2 + (n'\pi/\ln a)^2} \left(\begin{array}{c} \cos\frac{n'\pi\ln\rho}{\ln a} \\ \cos\frac{n'\pi\ln\rho}{\ln a}(-1)^{n'} \end{array} \right) . \end{aligned} \tag{3.6}$$

We find upon calculation that

$$\begin{aligned} G_N^{\mu\nu}(e^{i\phi}, e^{i\phi'}) &= G_N^{\mu\nu}(ae^{i\phi}, ae^{i\phi'}) \\ &= \alpha'(-2\alpha'\pi)^N \left(F^N\right)^{\mu\nu} \left(\frac{1}{1-a^{2m}}\right)^{N+1} \times \\ &\quad \times \left[(1+a^m)^{2N+2} + (1-a^m)^{2N+2}\right] \times \\ &\quad \times \sum_{m=1}^{\infty} \frac{1}{m} \left\{ \begin{array}{c} \sin m(\phi-\phi') \ N \text{ odd} \\ \cos m(\phi-\phi') \ N \text{ even} \end{array} \right\}, \end{aligned} \tag{3.7}$$

while

$$\begin{aligned} G_N^{\mu\nu}(e^{i\phi}, ae^{i\phi'}) &= G_N^{\mu\nu}(ae^{i\phi}, e^{i\phi'}) \\ &= \alpha'(-2\alpha'\pi)^N \left(F^N\right)^{\mu\nu} \left(\frac{1}{1-a^{2m}}\right)^{N+1} \times \\ &\quad \times \left[(1+a^m)^{2N+2} - (1-a^m)^{2N+2}\right] \times \\ &\quad \times \sum_{m=1}^{\infty} \frac{1}{m} \left\{ \begin{array}{c} \sin m(\phi-\phi') \ N \text{ odd} \\ \cos m(\phi-\phi') \ N \text{ even} \end{array} \right\}. \end{aligned} \tag{3.8}$$

The obvious generalization of (1.2) is to sum the commutators when the fields are at the same ends of the string with those where the fields are at opposite ends of the string, since the ends are on the same brane. In other words, to calculate

$$\begin{aligned} [X^\mu, X^\nu] = \frac{1}{4}\Big(& [X^\mu(1), X^\nu(1)] + [X^\mu(1), X^\nu(a)] + \\ & + [X^\mu(a), X^\nu(1)] + [X^\mu(a), X^\nu(a)] \Big) . \end{aligned} \tag{3.9}$$

We thus find for a fixed value of a that

$$[X^\mu, X^\nu] = \sum_{N\,\mathrm{odd}} \sum_{m=1}^{\infty} \lim_{\epsilon\to 0} (2\alpha')^{N+1}(-\pi)^N \left(F^N\right)^{\mu\nu} \left(\frac{1+a^m}{1-a^m}\right)^{N+1} \frac{2}{m}\sin m\epsilon$$
$$= \sum_{m=1}^{\infty} \lim_{\epsilon\to 0} 2\alpha' \left(\frac{-2\pi\alpha' F k^2}{1-(2\pi\alpha' F k)^2}\right)^{\mu\nu} \frac{2}{m}\sin m\epsilon\,, \tag{3.10}$$

where $k = \frac{1+a^m}{1-a^m}$.

Now, to calculate the full commutator, it is necessary to integrate over the Teichmuller parameter, with the measure as given in [13, 16], so

$$[X^\mu, X^\nu] = \int_0^1 \frac{da}{a} \sum_{m=1}^{\infty} \lim_{\epsilon\to 0} 2\alpha' \left(\frac{-2\pi\alpha' F k^2}{1-(2\pi\alpha' F k)^2}\right)^{\mu\nu} \frac{2}{m}\sin m\epsilon\,. \tag{3.11}$$

We note that $F_{\mu\nu}$ can be block diagonalized, and concentrate on one block: X^{2i}, X^{2i+1}. After trivial manipulations, we have

$$\left[X^{2i}, X^{2i+1}\right] = \int_0^1 \frac{da}{a} \sum_{m=1}^{\infty} \lim_{\epsilon\to 0} 2\alpha' \frac{2}{m}\sin m\epsilon \frac{\beta}{1+\beta^2} \frac{1+2a^m+a^{2m}}{1+a^{2m}-2a^m\left(\frac{1-\beta^2}{1+\beta^2}\right)}\,, \tag{3.12}$$

where $\beta = -2\pi\alpha' F$, and F is the field strength in this particular block. The m dependence in the integral can be accommodated by the change of variables, noting that $da/a = \frac{1}{m}\frac{d(a^m)}{a^m}$ and this gives

$$\left[X^{2i}, X^{2i+1}\right] = \sum_{m=1}^{\infty} \lim_{\epsilon\to 0} 2\alpha' \frac{2}{m}\sin m\epsilon \frac{\beta}{1+\beta^2}\frac{1}{m}\left[\ln x|_{x=0}^{x=1} + \frac{2}{\beta}\tan^{-1}\beta + \right.$$
$$\left. + \frac{2}{\beta}\tan^{-1}\frac{1-\beta^2}{2\beta}\right]. \tag{3.13}$$

Clearly, the only term that is divergent is the term $\ln x$, and if we change the original lower limit of integration to $\delta \to 0$ and so change the lower limit on x to δ^m, we obtain for this particular block

$$\left[X^{2i}, X^{2i+1}\right] = 2\pi\alpha' \frac{\beta}{1+\beta^2} \lim_{\delta\to 0} \ln\frac{1}{\delta}\,, \tag{3.14}$$

and since the other blocks will have the same structure, we find

$$[X^\mu, X^\nu] = 2\pi\alpha' \left(\frac{-2\pi\alpha' F}{1-(2\pi\alpha' F)^2}\right)^{\mu\nu} \lim_{\delta\to 0} \ln\frac{1}{\delta}\,. \tag{3.15}$$

The coefficient of the divergent part is exactly the same as in the case of the disk, and the divergence is interpreted [17] as coming from the open string tachyon.

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3.2 Annulus corrections: neutral string

The case where the two Wilson loops are oppositely oriented gives the neutral string. Because of this in eq. (3.5) we have $F_1 = -F_2 = F$ and the analog of eq. (3.6) is

$$G_N^{\mu\nu}(\rho e^{i\phi}, \rho' e^{i\phi'}) = (-2\alpha'\pi)^N \frac{2\alpha'}{\ln^2 a} \left(F^N\right)^{\mu\nu} \sum_{m=1}^{\infty} \frac{1}{m} \left\{ \begin{array}{c} \sin m(\phi-\phi')\ N \text{ odd} \\ \cos m(\phi-\phi')\ N \text{ even} \end{array} \right\} \times$$
$$\times \sum_n \frac{1}{m^2 + (n\pi/\ln a)^2} \left(\cos\frac{n\pi\ln\rho}{\ln a}, \cos\frac{n\pi\ln\rho}{\ln a}(-1)^n \right) \times$$
$$\times\, \Omega (G_m \Omega)^{N-1} \sum_{n'} \frac{1}{m^2 + (n'\pi/\ln a)^2} \times$$
$$\times \begin{pmatrix} \cos\frac{n'\pi\ln\rho}{\ln a} \\ \cos\frac{n'\pi\ln\rho}{\ln a}(-1)^{n'} \end{pmatrix}. \tag{3.16}$$

In the above, $\Omega = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, which is the field strength independent part of the interaction vertex. Upon diagonalizing $G_m\Omega$ we find that

$$G_N^{\mu\nu}(\rho e^{i\phi}, \rho' e^{i\phi'}) = (-2\alpha'\pi)^N \frac{2\alpha'}{\ln^2 a} \left(F^N\right)^{\mu\nu} \sum_{m=1}^{\infty} \frac{1}{m} \left\{ \begin{array}{c} \sin m(\phi-\phi')\ N \text{ odd} \\ \cos m(\phi-\phi')\ N \text{ even} \end{array} \right\} \times$$
$$\times \sum_n \frac{1}{m^2 + (n\pi/\ln a)^2} \left(\cos\frac{n\pi\ln\rho}{\ln a}, \cos\frac{n\pi\ln\rho}{\ln a}(-1)^n \right) \times$$
$$\times \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & -a^m \\ a^m & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & (-1)^{N-1} \end{pmatrix} \begin{pmatrix} 1 & -a^m \\ a^m & -1 \end{pmatrix} \times$$
$$\times \left(\frac{1}{1-a^{2m}} \right) \sum_{n'} \frac{1}{m^2 + (n'\pi/\ln a)^2} \begin{pmatrix} \cos\frac{n'\pi\ln\rho}{\ln a} \\ \cos\frac{n'\pi\ln\rho}{\ln a}(-1)^{n'} \end{pmatrix}. \tag{3.17}$$

As in the case of the charged string, since the ends of the string terminate on the same brane, it is necessary to sum the contribution of all possible commutators to determine the total commutator. However, the above equation has a structure that is easy to analyze: the commutator of crossover terms, like $[X^\mu(1), X^\nu(a)]$ explicitly vanish, and $[X^\mu(1), X^\nu(1)] = -[X^\mu(a), X^\nu(a)]$, so the annulus level commutator for a neutral string explicitly vanishes. In retrospect, this is not a surprising result. As shown above, for a charged bosonic string the commutation relation at annulus level consists of a constant, identical to that for the disk level calculation multiplied by the tachyon divergence. It is trivial to see that for an uncharged string there is no disk level contribution to the non-trivial commutator because the uncharged string does not couple to the background field, and we show that this property persists at one loop.

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3.3 Annulus corrections: distinct branes

We now examine the most general case, that of two distinct branes with independent background fields on each. Following the analysis of the previous sections, we find that the correction due to N interactions with the backgrounds is

$$
\begin{aligned}
G_N^{\mu\nu}(\rho e^{i\phi}, \rho' e^{i\phi'}) = (-2\alpha'\pi)^N \frac{2\alpha'}{\ln^2 a} \sum_{m=1}^{\infty} \frac{1}{m} \begin{Bmatrix} \sin m(\phi-\phi') \; N \text{ odd} \\ \cos m(\phi-\phi') \; N \text{ even} \end{Bmatrix} \times \\
\times \sum_n \frac{1}{m^2 + (n\pi/\ln a)^2} \left(\cos\frac{n\pi\ln\rho}{\ln a}, \cos\frac{n\pi\ln\rho}{\ln a}(-1)^n \right) \times \\
\times \Omega^{\mu\alpha_1} G_m \cdots G_m \Omega^{\alpha_{N-1}\nu} \times \\
\times \sum_{n'} \begin{pmatrix} \cos\frac{n'\pi\ln\rho}{\ln a} \frac{1}{m^2+(n'\pi/\ln a)^2} \\ \cos\frac{n'\pi\ln\rho}{\ln a} \frac{(-1)^{n'}}{m^2+(n'\pi/\ln a)^2} \end{pmatrix} .
\end{aligned}
\tag{3.18}
$$

Since the two-branes are distinct, it is only sensible to consider the commutator between X fields at the same end of the string. Explicitly we find that

$$
\begin{aligned}
G_N^{\mu\nu}(e^{i\phi}, e^{i\phi'}) = 2\alpha'(-2\alpha'\pi)^N \sum_{m=1}^{\infty} \frac{1}{m} \begin{Bmatrix} \sin m(\phi-\phi') \; N \text{ odd} \\ \cos m(\phi-\phi') \; N \text{ even} \end{Bmatrix} \times \\
\times \sum_n \Big(\coth m\ln a, \operatorname{csch} m\ln a \Big) \Omega^{\mu\alpha_1} G_m \cdots \times \\
\times G_m \Omega^{\alpha_{N-1}\nu} \begin{pmatrix} \coth m\ln a \\ \operatorname{csch} m\ln a \end{pmatrix} .
\end{aligned}
\tag{3.19}
$$

We consider this in the limit of $a \to 0$, which corresponds to the disk amplitude. In this limit, we have

$$
\begin{aligned}
G_N^{\mu\nu}(e^{i\phi}, e^{i\phi'}) = (2\alpha')^{N+1}(-\pi)^N (F_1^N)^{\mu\nu} \times \\
\times \sum_{m=1}^{\infty} \frac{1}{m} \begin{Bmatrix} \sin m(\phi-\phi') \; N \text{ odd} \\ \cos m(\phi-\phi') \; N \text{ even} \end{Bmatrix} ,
\end{aligned}
\tag{3.20}
$$

so on the first brane,

$$
[X^\mu, X^\nu] = 2\pi\alpha' \left(\frac{-2\pi\alpha' F_1}{1-(2\pi\alpha' F_1)^2} \right)^{\mu\nu} . \tag{3.21}
$$

Similarly, it is clear that in this limit on the second brane we have

$$
[X^\mu, X^\nu] = 2\pi\alpha' \left(\frac{-2\pi\alpha' F_2}{1-(2\pi\alpha' F_2)^2} \right)^{\mu\nu} . \tag{3.22}
$$

The cross terms are clearly zero. This result reproduces and generalizes that found in [6] for noncommutativity on the ends of branes. It is important to note that this result does not rely on any relations between the two fields on the branes, in particular, they do not need to have the same block diagonal form.

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4. Conclusion

In this paper we have shown a method for calculating the disk and annulus contributions to non-commutative geometry. In particular, we have shown that the contribution from the higher order annulus diagrams is that which comes from the disk, multiplied by the tachyon divergence. The benefits of the method presented are that it does not depend upon any particular properties of the background, other than the fact it is constant.

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